Shaded Region Solutions

1. Let the radius of the circle be r, then the side of equilateral triangle will be $2r$.

   Area of the shaded region equals to area of the equilateral triangle minus area of three 60 degrees sectors.

   Area of a 60 degree sector is 1/6 of the are of whole circle (as whole circle is 360 degrees and 60 is 1/6 of it), hence are of 3 such sectors will be 3/6=1/2 of the area of whole circle, so $area_{sectors}=\frac{\pi r^2}{2}$ (here if you could spot that $\frac{\pi r^2}{2}$ should correspond to 32π then you can write $\frac{\pi r^2}{2}=32\pi$ --> $r=8$);

   Area of equilateral triangle equals to $a^2\frac{\sqrt{3}}{4}$, where a is the length of a side. So in our case $area_{equilateral}=(2r)^2*\frac{\sqrt{3}}{4}=r^2\sqrt{3}$;

   Area of the shaded region equals to $64\sqrt{3}-32\pi$, so $area_{equilateral}-area_{sectors}=r^2\sqrt{3}-\frac{\pi r^2}{2}=64\sqrt{3}-32\pi$ --> $r^2=\frac{2(64\sqrt{3}-32\pi)}{2\sqrt{3}-\pi}=\frac{64(2\sqrt{3}-\pi)}{(2\sqrt{3}-\pi)}=64$ --> $r=8$.

   Answer: B.

2. Say the length and the width of the picture are x and y respectively. Since they have the same ratio as the lenght and width of the frame, then $\frac{x}{y}=\frac{18}{15}$ --> $y=\frac{5}{6}x$.

   Next, since the frame encloses a rectangular picture that has the same area as the frame itself and the whole area is $18*15$, then the areas of the frame (shaded region) and the picture (inner region) are $\frac{18*15}{2}=9*15$ each.

   The area of the picture is $xy=9*15$ --> $x*(\frac{5}{6}x)=9*15$ --> $x^2=2*81$ --> $x=9\sqrt{2}$.

   Answer: A.

3. The area of the shaded region is $area_{shaded}=\pi R^2-\pi r^2$ and the area of the smaller circle is $area_{small}=\pi r^2$. Given: $\pi R^2-\pi r^2=3\pi r^2$ --> $R^2=4r^2$ --> $R=2r$;

   Now, the ratio of the circumference of the larger circle to the that of the smaller circle is $\frac{C}{c}=\frac{2\pi R}{2\pi r}=\frac{2r}{r}=2$.

Answer: C.

4. We can do this by calculating the area of the trapezoid as well.

   First of all the angle = 120 degrees. Non shaded area will consist of the areas of 6*120 degrees sectors plus one whole circle = 3 whole circles = $3\pi r^2=27\pi$, (as $r=3$, half of the side of the hexagon).

   Now if we want to proceed with the trapezoid areas. We would have two trapezoids. Bases would be $a=6$ (the smallest base) and $b=12$ (the largest base). $Area=\frac{a+b}{2}*h$. The height of the trapezoid can be calculated as the side in 30-60-90 right triangle. Height would be the opposite side to the 60 degrees angle (half of 120 degrees). In 30-60-90 right triangle the ratio of the sides is $1/\sqrt{3}/2$, the height would correspond with \sqrt{3} and the hypotenuse (which on the other hand would be the side of the hexagon=6) would correspond with 2. so we'll have $2/\sqrt{3}=\frac{6}{h}$ --> $h=3\sqrt{3}$. Area of the trapezoid would be $Area=\frac{a+b}{2}*h=\frac{6+12}{2}*3\sqrt{3}=27\sqrt{3}$. As we have two trapezoids the whole area would be $2*27\sqrt{3}=54\sqrt{3}$.

   Area of the shaded region $54\sqrt{3}-27\pi$.

   Answer: E.

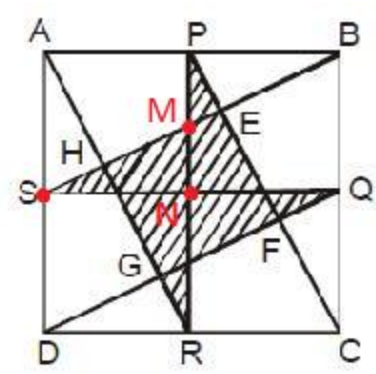
5. The dimensions of the rectangle is 5 radii by 2 radii, thus 35 by 14. Therefore its area is 35*14.

   The area of the two circles and the semicircle is $2.5*\pi r^2=\frac{5}{2}*\frac{22}{7}*7^2=35*11$.

   Thus the area of the shaded region is approximately 35*14-35*11 = 35(14-11)=35*3=105.

   Answer: A.

6. Consider square APNS and say its side is 1. In this case:

   The area of APNS is 1.
   MN = 1/2, which means that the area of SMN is 1/2*1/2*1=1/4.

   (shaded)/(square)=(1/4)/1=1/4.

   The ratio for the entire square would be the same.

   Answer: B.

7. The area of equilateral triangle is $side^2*\frac{\sqrt{3}}{4}$.

   Given that $side^2*\frac{\sqrt{3}}{4}=3$ --> $side=2\sqrt[4]{3}$.

   Since E is the midpoint of AC, then CE = radius = half of the side = $\sqrt[4]{3}$.

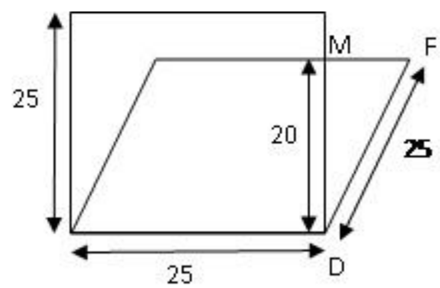
   Next, as ABC is an equilateral triangle, then angle C is 60 degrees, thus sector CED is 1/6 of the area of the whole circle --> $area=\pi*(\sqrt[4]{3})^2*\frac{1}{6}=\frac{\pi\sqrt{3}}{6}=\frac{\pi}{2\sqrt{3}}$.

   The area of the shaded region = $3-\frac{\pi}{2\sqrt{3}}$.

   Answer: D.

8. Area of square = 625 so side = 25
   Area of rhombus = 500. So altitude = 500/25 = 20
   MF = root (25^2 - 20^2) = 15
   Area of triangle MFD = (1/2) * 15 * 20 = 150

   Area of shaded region = 625 - (500 - 150) = 275



9. The whole area = 16T.

   The areas of three unshaded triangles are = T + 6T + 4T = 11T (half of a rectangle with area of 2 + half of a rectangle with area of 12 + half of a rectangle with area of 8).

   The area of the shaded region = 16T - 11T = 5T.

   Answer: B.

10. The Central Angle Theorem states that the measure of inscribed angle is always half the measure of the central angle.

    Therefore the angle O is twice the angle C, or 90 degrees.

    Next, since OA=OB=radius, then triangle OAB is isosceles right triangle, or 45-45-90 triangle, and thus its sides are in ratio $1:1:\sqrt{2}$. Therefore $r+r+r\sqrt{2}=8+\sqrt{32}$ --> $r(2+\sqrt{2})=4(2+\sqrt{2})$ --> $r=4$.

    The area of the circle is $\pi r^2=16\pi$ and the area of the sector OAB is 1/4 of that, or 4π.

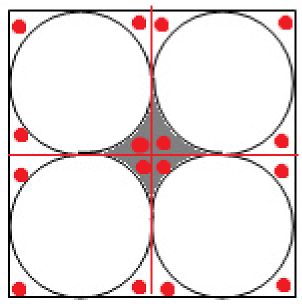
    The area of triangle OAB is $\frac{1}{2}*4*4=8$, therefore the area of shaded region is $4\pi-8$.

Answer: A.

11. The areas of regions with red dots are equal. So, we have 16 equal regions and we need the area of four of them. The area of all 16 is equal to the area of the square minus the area of four circles.

The area of the square = $20^2 = 400$.

The area of four circles = $4*(\pi r^2) = 4*(\pi 5^2) = 100\pi$ (the diameter of each circle is 1/2 of the side, thus the radius of each circle is 1/4 of the side).



The area of 16 regions = $400 - 100\pi$.

The area of shaded region (4 regions with red dots) = $\frac{400-100\pi}{4} = 100 - 25\pi$.

Answer: C.

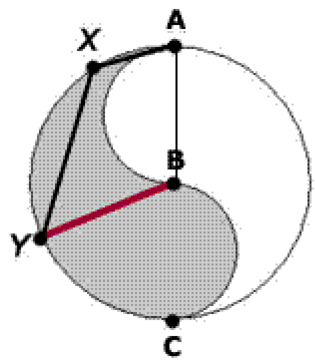
Else you could simply find the area of the smaller square (1/4 of the bigger) and subtract the area of the circle. This way you'd also get the area of 4 regions with red dots.

12. According to the central angle theorem <ABY=2*(180-105)=150 (for more on this check Circles chapter of Math Book: math-circles-87957.html). Hence <CBY=180-150=30.

The area of sector $ABY = \frac{150}{360} * \pi r^2 = \frac{5}{12} \pi r^2$;

The area of sector $CBY = \frac{30}{360} * \pi r^2 = \frac{1}{12} \pi r^2$;

The area of each of two small semicircles is $\frac{\pi(\frac{r}{2})^2}{2} = \pi \frac{r^2}{8}$ (as its radius is half of the radius of the big circle);



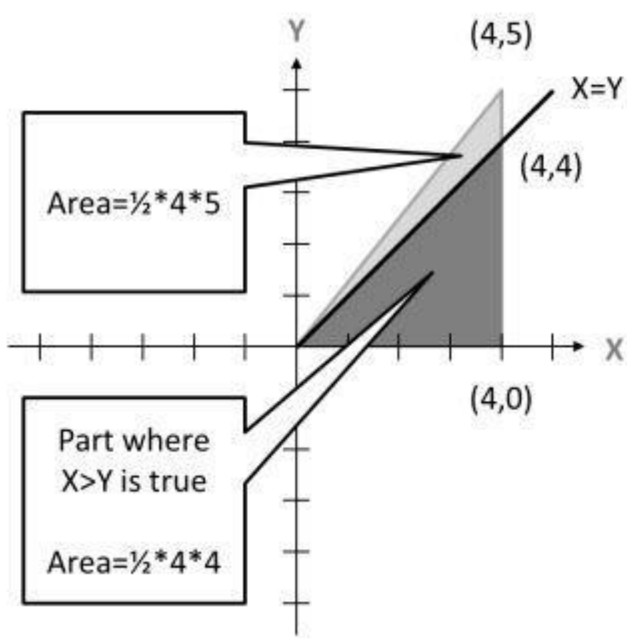
The area of the shaded region above BY is $\frac{5}{12}\pi r^2 - \pi\frac{r^2}{8} = \frac{7}{24}\pi r^2$;

The area of the shaded region below BY is $\frac{1}{12}\pi r^2 + \pi\frac{r^2}{8} = \frac{5}{24}\pi r^2$;

Ratio of the areas of the shaded regions is $\frac{7}{5}$.

Answer: D.

13. We have right triangle with the area=4*5/2=10. Consider the line y<x. All the points which satisfy this equation (are below the line y=x) and lie in the triangular region obviously will have x more than y, which is exactly what we want (as x>y --> x-y>0).



The probability that the point will be from this region is: Area of this region/Area of the triangle.

Favorable region is also right triangle with vertexes at (0,0) (4,0) and (4,4). As y=x intersects the side of our original triangle at the point (4,4). You''ll see it easily if you draw it. So favorable are=4*4/2=8.

P=8/10=4/5

Answer: E.

14. Given that $\frac{xy}{2}=24$ and $x=y+2$ --> $xy=48$ --> $(y+2)y=48$.

$z^2=x^2+y^2$ --> $z^2=(y+2)^2+y^2=2y^2+4y+4=2y(y+2)+4$.

Since $(y+2)y=48$, then $z^2=2y(y+2)+4=2*48+4=100$ --> $z=10$.

Answer: E.

15. Property: In two similar triangles, the ratio of their areas is the square of the ratio of their sides: $\frac{AREA}{area}=\frac{S^2}{s^2}$.

As both big and inscribed triangles are equilateral then they are similar, so $\frac{AREA}{area}=\frac{S^2}{s^2}=\frac{2^2}{1^2}=4$, so if $AREA=K$ then $area=\frac{K}{4}$ --> the area of the shaded region equals to $area_{shaded}=K-\frac{K}{4}=\frac{3K}{4}$.

Answer: A.

16. Since the area of unshaded region is equal to the are of shaded region, then the area of the big triangle is twice the area of the little triangle (unshaded region): $\frac{AREA_{ABC}}{area_{MNP}}=\frac{2}{1}$

Next, triangles ABC and MNP are similar. In two similar triangles, the ratio of their areas is the square of the ratio of their sides: $\frac{AREA}{area}=\frac{S^2}{s^2}$. Thus $\frac{AREA_{ABC}}{area_{MNP}}=\frac{AC^2}{MP^2}$ --> $\frac{2}{1}=\frac{7^2}{MP^2}$ --> $MP^2=\frac{7^2}{2}$ --> $MP=\frac{7}{\sqrt{2}}=\frac{7\sqrt{2}}{2}$.

Answer: D.

17. The area of the big circle is $\pi R^2=324\pi$;
The area of the small circle (the unshaded region) is πr^2;

The area of the shaded region is $324\pi-\pi r^2$;

We are told that the area of the shaded region is equal to the area of the unshaded region, so $\pi r^2=324\pi-\pi r^2$ --> $2\pi r^2=324\pi$ --> $r^2=162$ --> $r=9\sqrt{2}$.

Answer: D.

18. The legs of right isosceles triangles FAE and GCH equal to half of the side of the square, so to 4/2=2, hence their combined area is $2*(\frac{1}{2}*2*2)=4$;

The same way the radii of arcs FG and EH equal to 4/2=2. Since both arcs are 90 degrees, then their commbined area is $2*\frac{90}{360}*\pi*r^2=2\pi$;

The are of the square is 4^2=16, thus the area of the shaded region is $16-(4+2\pi)=2(6-\pi)$.

Answer: D.

19. The area of an equilateral triangle is $area=a^2*\frac{\sqrt{3}}{4}$, where a is the length of a side. Thus $a^2*\frac{\sqrt{3}}{4}=6$ --> $a^2=8*\sqrt{3}$ --> radius of the circles (which make arcs) is a/2, so $r^2=\frac{a^2}{4}$. Now, as the triangle is equilateral each arc is 60 degrees, so the area of white region (non-shaded) is $\frac{60*3}{360}*\pi r^2=\frac{1}{2}*\pi\frac{8*\sqrt{3}}{4}=\pi\sqrt{3}$ (three 60 degrees arcs).

The area of the shaded region is $6-\pi\sqrt{3}$.

Answer: D.

20. The area of a rectangle equals to $area=width*length$;

The area of a triangle equals to $area=\frac{1}{2}*base*height$ (also the area of a square=diagonal^2/2);

Now it's easy to calculate the areas of given figure:
$area_P=\frac{1}{2}*4*1=2$;
$area_Q=1*2=2$;
$area_R=\frac{2^2}{2}=2$;
$area_S=\frac{1}{2}*3*1=1.5$.

Answer: D.

21. Property of median: Each median divides the triangle into two smaller triangles which have the same area.

    So, AD is median and thus $y=z$, (we could derive this even not knowing the above property. Given: $area_{ABC}=\frac{x(y+z)}{2}=2*area_{ABD}=2*\frac{xy}{2}$ --> $y=z$).

    Next: AD is hypotenuse in right triangle ABD and thus $AD=\sqrt{x^2+y^2}=\sqrt{x^2+4y^2-3y^2}=\sqrt{x^2+(2y)^2-3y^2}=\sqrt{w^2-3y^2}$ (as $2y=y+z=BC$ and $AC^2=w^2=AB^2+BC^2=x^2+(2y)^2$, so $w^2=x^2+(2y)^2$).

    Answer: D.

22. As the garden is square then the base must also equal to 9, so the middle piece of base equals to $9-3-x=6-x$.

    As area of the shaded region equals to the area of unshaded region then: $18+6x+3*(6-x)=9+2*3(6-x)+3x$ --> $x=1.5$.

    Answer: C

23. Method 1:

    Area of ABCD is 1.
    Area of PQRS = (1/2)*(diagonal1)*(diagonal2)
    Both diagonals of PQRS are 1 each since they are the same lengths as sides of ABCD
    Area of PQRS = (1/2)*1*1 = 1/2

    Area of leftover region = 1 - 1/2 = 1/2
    The leftover region after you cut out PQRS is split into 8 equal areas and the red region is one of those 8.
    Hence area of red region is (1/8)*(1/2) = 1/16

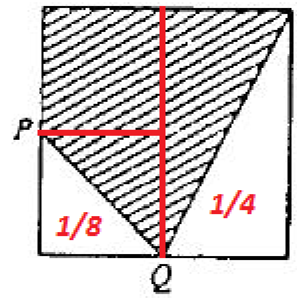
    Method 2:

    The perimeter of ABCD is 4 so each side is 1. So each half side is 1/2.
    In triangle APQ, AP and AQ are 1/2 each so $PQ=1/\sqrt{2}$ (using Pythagorean theorem)
    So half of PQ is $1/2\sqrt{2}$

    Area of red triangle = $(1/2)*(1/2\sqrt{2})*(1/2\sqrt{2})=1/16$

24. (1) The area of the square region is 36. We don't know positions of P and Q. Not sufficient.

(2) P and Q are midpoints of two sides of the square as shown. Look at the diagram below:
The shaded region is 1-1/4-1/8=5/8. Sufficient.
Answer: B.



25. (1) $PR=\frac{4\sqrt{10}}{3}$. We know two sides (PR and PQ) in right triangle PQR, thus we can find the third side PQ. Sufficient.

(2) The ratio of the area of the unshaded region to the total area of the shaded region is 2 to 1. Say LM = PQ = x, then the area of the shaded region is 2*(1/2*4*x)=4x. The area of unshaded region is 4*4-4x=16-4x. Thus we have that (unshaded)/(shaded)=(16-4x)/4x=2/1. We can find x. Sufficient.

Answer: D.

26. Actually there is no need for any calculations. From both statements we have everything "fixed", only one case possible where we know everything possible.

(1) The radius of the circle with center O is 4 --> diameter = side = 8. We have "fixed" circle and square. We can find anything there. Sufficient.

(2) The area of triangle ADC is 32 --> 1/2*AD^2=32 --> AD=8 --> AD = diameter = 8. The same info as above. Sufficient.

Answer: D.

27. In equilateral triangle all angles equal to 60 degrees and $Area_{equilateral}=\frac{a^2\sqrt{3}}{4}$, where a is the length of a side.

(1) Angle DFE is 90º --> angles ABF and BDC must also be 90º (for example $\measuredangle BDC=180-\measuredangle BDF-\measuredangle EDF=180-60-30$ and the same for ABF). Also as $\measuredangle DEF=\measuredangle BCD=\measuredangle BAF=60$, then triangles DFE, BCD and BAF are 30-60-90 trianlges. In such triangle sides are in the ratio: $1:\sqrt{3}:2$ (smallest side (1) is opposite the smallest angle (30°), and the longest side (2) is opposite the largest angle (90°)).

So if $DE=2x$ (hypotenuse in right triangle DFE), then $DC=x$ (smaller leg in right triangle BCD) and $BD=\sqrt{3}x$ (larger leg in right triangle BCD, also the side of inscribed triangle). So the side of triangle ACE would be $CE=DC+DE=x+2x=3x$

Area of the shaded region (right triangle BDC) would be $Area_{BDC}=\frac{BD*DC}{2}=\frac{\sqrt{3}x*x}{2}=\frac{\sqrt{3}x^2}{2}$ and the area of equilateral triangle $Area_{ACE}=\frac{a^2\sqrt{3}}{4}=\frac{(3x)^2*\sqrt{3}}{4}=\frac{9x^2\sqrt{3}}{4}$;

$\frac{Area_{BDC}}{Area_{ACE}}=\frac{\sqrt{3}x^2}{2}*\frac{4}{9x^2\sqrt{3}}=\frac{2}{9}$.

Sufficient.

(2) The length of AF is $10*\sqrt{3}$. Multiple breakdowns are possible, hence multiple ratios of areas. Not sufficient.

Answer: A.